Question 1. A manufacturer offers a warranty paying 1000 at the time of failure for each machine that fails within 5 years of purchase. One customer purchases 500 machines. The manufacturer wants to establish a fund for warranty claims from this customer. The manufacturer wants to have at least a 95% probability that the fund is sufficient to pay the warranty claims.

You are given:

(i) The constant force of failure for each machine is $\mu = 0.02$

(ii) The force of interest is $\delta = 0.02$.

(iii) The times until failure of the machines are independent.

Using the normal approximation, determine the minimum size of the fund.

Question 2. You are given :

(i) The force of interest is constant: $\delta = 0.06$

(ii) The force of mortality is constant: $\mu(x) = 0.04$ for all $x \geq 0$.

Calculate the standard deviation of $\bar{a}_{\overline{T|}}$

Question 3. Consider a whole life annuity payable continuously at the rate of 1 per year. Let $Y = \bar{a}_{\overline{T|}}$ be the present value random variable. The uniform distribution of deaths over each year of age is assumed. On the basis of the Illustrative Life Table and $\delta = 0.1$, determine the median of the present value random variable.

Question 4. A fund is established to pay annuities to 100 independent lives age x. Each annuitant will receive 10,000 per year continuously until death. You are given:

(i) $\delta = 0.06$

(ii) $\overline{A}_x = 0.40$

(iii) ${}^2\overline{A}_x = 0.25$

Calculate the amount (in millions) needed in the fund so that the probability, using the normal approximation, is 0.90 that the fund will be sufficient to provide the payments.

Question 5. For an annuity payable semiannually, you are given:

(i) Deaths are uniformly distributed over each year of age.

(ii) $q_{69} = 0.03$

(iii) $i = 0.06$

(iv) $1000\overline{A}_{70} = 530$

Calculate $\ddot{a}^{(2)}_{69}$.

Question 6. Using the assumption of a uniform distribution of deaths in each year of age and the Illustrative Life Table with interest at the effective annual rate of 6%, calculate $\ddot{a}^{(2)}_{40:\overline{30|}}$.

Question 7. You are given:

(i) $\ell_x = 100,000(100 - x), 0 \le x \le 100$

(ii) $i = 0$

Calculate $(I_{\overline{2|}}\overline{a})_{95}$ exactly.

Question 8.

(i) Mortality follows the Illustrative Life Table.
(ii) $\cancel{i = 0.06}$
(iii) Assume that the function $v^{k+j/m}\ {}_{k+j/m}p_x$ is linear in j for $j = 0, 1, \cdots, m-1$.
(iv) \$1,000 monthly payment at the beginning ~~of each month from~~ age 60 to 70
(v) \$2,000 monthly payment at the beginning of each month from age 70 to 80.

Determine the actuarial accumulated value of the annuity at age 80.

Question 9.

(i) Mortality follows the Illustrative Life Table.
(ii) $i = 0.06$
(iii) Consider an 10−year term insurance payable at the end of the year of death of (20), under which the death benefit in case of death in year $k+1$ is $\ddot{s}_{\overline{k+1|}}$, $0 \le k < 10$.

Determine the actuarial present value.

ACT 3130 Actuarial Models 1
Test 3 Solution

1.

$$\overline{A}^{1}_{x:\overline{5}|} = \frac{\mu}{\mu+\delta}(1 - v^5\,{}_5p_x) = \frac{0.02}{0.04}(1 - e^{-0.04\times 5}) = 0.0906$$
$${}^2\overline{A}^{1}_{x:\overline{5}|} = \frac{\mu}{\mu+2\delta}(1 - v^{10}\,{}_5p_x) = \frac{0.02}{0.06}(1 - e^{-0.06\times 5}) = 0.0864$$
$$\mathrm{E}(S) = 500 \times 1000 \times 0.0906 = 45300$$
$$\mathrm{Var}(S) = 500 \times 1000^2 \times [0.0864 - (0.0906)^2] = 39,095,820 = (6,252.67)^2$$
$$\Rightarrow \quad F = 45,300 + 1.645 \times 6,252.67 = 55,586$$

2. When both δ and μ are constant, both $\overline{A}_x$ and $\overline{a}_x$ become greatly simplified. Know these formulas for the exam.

$$\overline{A}_x = \frac{\mu}{\mu+\delta} = \frac{0.04}{0.1} = 0.4$$
$${}^2\overline{A}_x = \frac{\mu}{\mu+2\delta} = \frac{0.04}{0.16} = 0.25$$
$$\mathrm{Var}(\overline{a}_{\overline{T}|}) = \mathrm{Var}\left(\frac{1-v^T}{\delta}\right) = \frac{{}^2\overline{A}_x - (\overline{A}_x)^2}{\delta^2} = \frac{0.25 - 0.4^2}{0.06^2} = 25 = 5^2$$

The standard deviation is 5.

3. We need to find $\xi_Y^{0.5}$ which satisfies $F_Y(\xi_Y^{0.5}) = 0.5$. We have

$$F_Y(\xi_Y^{0.5}) = F_T\left[\frac{-\log(1-\delta\xi_Y^{0.5})}{\delta}\right].$$

Thus we will first find $\xi_T^{0.5}$ which satisfies

$$\frac{\ell_{30+\xi_T^{0.5}}}{\ell_{30}} = 0.5.$$

From the Illustrate Life Table we have $\ell_{30}(0.5) = 4,750,690.5$, $\ell_{77} = 4,828,182$ and $\ell_{78} = 4,530,360$. Under the UDD (over each year of age) assumption, we have

$$\xi_Y^{0.5} = 47 + \frac{4,750,690.5 - 4,530,360}{4,828,182 - 4,530,360} = 47.74.$$

Now solve

$$\xi_Y^{0.5} = \frac{1 - e^{-\delta\xi_T^{0.5}}}{\delta} = 9.92.$$

4.

$$\overline{a}_x = \frac{1-\overline{A}_x}{\delta} = \frac{1-0.40}{0.06} = 10$$
$$\mathrm{Var}(\overline{a}_{\overline{T(x)}|}) = \frac{{}^2\overline{A}_x - (\overline{A}_x)^2}{\delta^2} = \frac{0.25 - 0.40^2}{0.06^2} = 25$$
$$\mathrm{E}(S) = 100 \times 10,000 \times 10 = 10,000,000$$
$$\mathrm{Var}(S) = 100 \times 10,000^2 \times 25 = 2.5 \times 10^{11}$$
$$\sigma(S) = \sqrt{2.5 \times 10^{11}} = 500,000$$
$$F = 10,000,000 + 1.282 \times 500,000 = 10,641,000$$

5.

$$
\begin{aligned}
A_{70} &= \frac{\delta}{i}\bar{A}_{70} = \frac{\ln(1.06)}{0.06}(0.53) = 0.5147 \\
\ddot{a}_{70} &= \frac{1-A_{70}}{d} = \frac{1-0.5147}{0.06/1.06} = 8.5736 \\
\ddot{a}_{69} &= 1 + vp_{69}\ddot{a}_{70} = 1 + \left(\frac{0.97}{1.06}\right)(8.5736) = 8.8457 \\
\ddot{a}_{69}^{(2)} &= \alpha(2)\ddot{a}_{69} - \beta(2) = (1.00021)(8.8457) - 0.25739 = 8.5902
\end{aligned}
$$

Note that $\alpha(2)$ and $\beta(2)$ are given in *Tables M.*

6. Using *Tables M,* we have

$$
\begin{aligned}
\alpha(2) &= 1.00021 \\
\beta(2) &= 0.25739 \\
{}_{30}E_{40} &= v^{30}\,{}_{30}p_{40} = 1.06^{-30}\frac{l_{70}}{l_{40}} = 1.06^{-30}\frac{6,616,155}{9,313,166} = 0.12369 \\
\ddot{a}_{40} &= 14.8166 \\
\ddot{a}_{70} &= 8.5693 \\
\ddot{a}_{40:30}^{(2)} &= \alpha(2)(\ddot{a}_{40} - {}_{30}E_{40}\,\ddot{a}_{70}) - \beta(2)(1 - {}_{30}E_{40}) \\
&= 1.00021(14.8166 - 8.5693 \times 0.1236894) - 0.25739(1 - 0.12369) \approx 13.53
\end{aligned}
$$

7. $(I_{\overline{2|}}\,\bar{a})_{95} = \int_0^5 {}_tp_x\,\mathrm{d}t + \int_1^5 {}_tp_x\,\mathrm{d}t = \int_0^5 \frac{5-t}{5}\,\mathrm{d}t + \int_1^5 \frac{5-t}{5}\,\mathrm{d}t = 4.1$

8. This is similar to AM Exercise 3.33.

$$
\begin{aligned}
\frac{m-1}{2m} &= \frac{11}{24} = 0.45833 \\
\ddot{a}_{80}^{(12)} &= 5.905 - 0.45833 = 5.4467 \\
\ddot{a}_{60:\overline{20|}}^{(12)} &= \ddot{a}_{60}^{(12)} - {}_{20}E_{60}\ddot{a}_{80}^{(12)} = (11.1454 - 0.45833) - (0.14906)(5.4467) = 9.87519 \\
\ddot{a}_{70:\overline{10|}}^{(12)} &= \ddot{a}_{70}^{(12)} - {}_{10}E_{70}\ddot{a}_{80}^{(12)} = (8.5693 - 0.45833) - (0.33037)(5.4467) = 6.31155 \\
\text{AAV} &= 12,000\left[\frac{\ddot{a}_{60:\overline{20|}}^{(12)}}{{}_{20}E_{60}} + \frac{\ddot{a}_{70:\overline{10|}}^{(12)}}{{}_{10}E_{70}}\right] = 12,000\left(\frac{9.87519}{0.14906} + \frac{6.31155}{0.33037}\right) = 1,024,254
\end{aligned}
$$

9. This is from AM Exercise 3.35.

$$
\begin{aligned}
\text{APV} &= \ddot{a}_{20:\overline{10|}} - {}_{10}p_{20}\,\ddot{a}_{\overline{10|}} = \ddot{a}_{20} - {}_{10}E_{20}\,\ddot{a}_{30} - {}_{10}p_{20}\,\ddot{a}_{\overline{10|}} \\
&= 16.5133 - 0.55164(15.8561) - \frac{9,501,381}{9,617,802}\frac{1-v^{10}}{d} = 0.05919
\end{aligned}
$$